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International Mathematics Competition (TIMC 2016)

Chiang Mai, Thailand 14 - 20 August 2016

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and contestant number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, electronic devices or protractor are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

Team: _____ ***Name:*** _____ ***No.:*** _____ ***Score:*** _____

For Juries Use Only

No.	Section A												Section B			Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3		
Score																	
Score																	

Section A.
In this section, there are 12 questions, each correct answer is worth 5 points. Fill in your answer in the space provided at the end of each question.

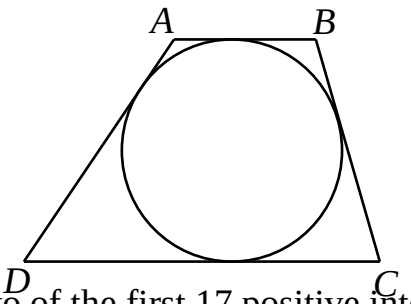
1. The cafeteria puts 289 pieces of bread out for the students every day. During a week, the students eat a different number of pieces each day. On some days, some pieces are left over. On other days, the cafeteria puts out more pieces until the students are full. The number of pieces left over is recorded as a positive integer. The number of extra pieces put out is recorded as a negative integer. The product of these seven numbers is -252. What is the total number of pieces of bread eaten during this week?

Answer : _____

2. Find the largest integer x for which there exists a positive integer y such that $2^{2x} - 3^{2y} = 55$.

Answer : _____

3. A circle of radius 12 cm touches all four sides of a quadrilateral *ABCD* with *AB* parallel to *DC*. If *BC* = 25 cm and the area of *ABCD* is 648 cm^2, determine the length, in cm, of *DA*.

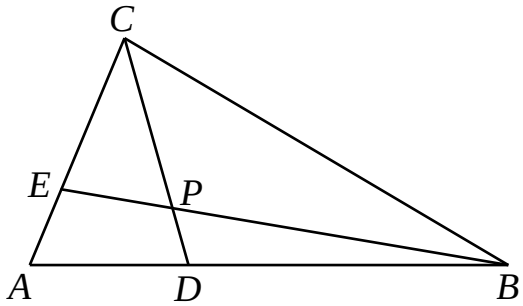


Answer : _____ cm

4. The product of two of the first 17 positive integers is equal to the sum of the other 15 numbers. What is the sum of these two numbers?

Answer : _____

5. *D* is a point on side *AB* and *E* is a point on side *AC* of triangle *ABC*. *P* is the point of intersection of *BE* and *CD*. The area of triangle *ABC* is 12 cm^2. Triangle *BPD*, triangle *CPE* and the quadrilateral *ADPE* all have the same area. What is the area, in cm^2, of *ADPE*?



Answer : _____ cm^2

6. Find the sum of the digits of the product of a number consisting of 2016 digits all of which are 6s, and another number consisting of 2016 digits all of which

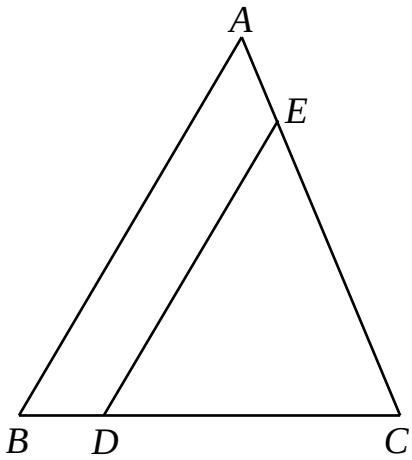
are 9s.

Answer : _____

7. Let n be a positive integer. Each of Tom and Jerry has some coins. If Tom gives n coins to Jerry, then Jerry will have 2 times as many coins as Tom. If instead Jerry gives 2 coins to Tom, then Tom will have n times as many coins as Jerry. Find the sum of all possible values of n.

Answer : _____

8. In triangle ABC, $BC = 13$ cm, $CA = 14$ cm and $BA = 15$ cm. D and E are points on sides BC and AC, respectively, such that DE is parallel to AB. If triangle EDC has the same perimeter as the quadrilateral $ABDE$, determine $\dfrac{BD}{DC}$.



Answer : _____

9. Three parallel lines L_1, L_2 and L_3 are such that L_1 is 1 cm above L_2 and L_3 is 2 cm below L_2. A right isosceles triangle has one of its vertices on each line. What is the sum, in cm^2, of all possible values for the area of this triangle?

Answer : _____

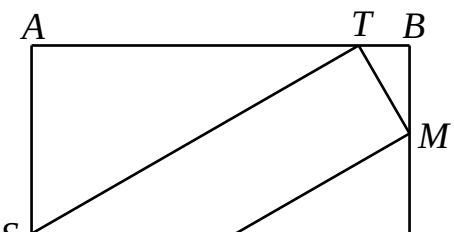
10. When a person with IQ 104 moved from village A to village B, the average IQ of both villages increased by 1. The sum of the population of the two villages is a prime number and the sum of the IQ of all people in both villages is 6102. Find the sum of the IQ of the people of village B including the new arrival.

Answer : _____

11. Alice is at the origin (0, 0) of the coordinate plane. She moves according to the roll of a standard cubical die. If the die rolls is 1, she moves 1 space to the right. If the die rolls is 2 or 3, she moves 1 space to the left. If the die rolls is 4, 5 or 6, she moves 1 space up. What is the probability that after four moves, Alice lands on the point (1, 1) for the first time?

Answer : _____

12. The diagram below shows a square $ABCD$ of side length 234 cm. The rectangles $CDPQ$ and $MNST$ are congruent. Find the length, in cm, of CM.



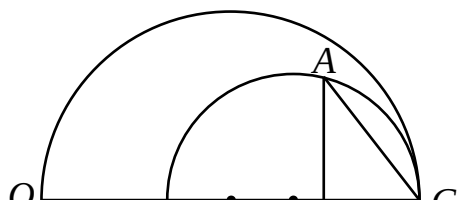
Answer : _____ cm

Section B.
Answer the following 3 questions, each question is worth 20 points. Show your detailed solution in the space provided.

1. Let a, b and c be positive real numbers such that $\frac{8a^2}{a^2+9}=b$, $\frac{10b^2}{b^2+16}=c$ and $\frac{6c^2}{c^2+25}=a$. Find $a+b+c$.

Answer : _____

2. R is a point on a segment CQ with CR=4 cm. A line perpendicular to CQ intersect the circles with diameters CR and CQ at A and B respectively, with A and B on opposite sides of CQ. If the circumradius of triangle ABC is $\sqrt{6}$ cm, find the length, in cm, of CQ.



Answer : _____ cm

3. What is the largest integer $n < 999$ such that $(n-1)^2$ divides $n^{2016} - 1$?

Answer : _____